$e^x =$

Birzeit University

Mathematics Department

Math 235 - Second Exam
First Semester 2013/2014

44.5

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Question 1. *(12 points)* **.Circle the most correct answer:**

1. *If* $y = 1 - 4e^{-3x}$*, then* $y''(x) =$ $+4e^{-3x}(+3) = (12e^{-3x})' \Rightarrow 12e^{-3x}(-3)$ $= -36e^{-3x}$

   (a) $y''(x) = 36e^{-3x}$ $-4e^{-3x} \cdot -3$

   [✓ (b)] $y''(x) = -36e^{-3x}$

   (c) $y''(x) = 48e^{-3x}$

   (d) $y''(x) = -48e^{-3x}$

2. *suppose that the marginal revenue(in dollars) for a certain product is* $\overline{MR} = 20 - 0.02x$*. then the total revenue from the sale of 100 units.*

   $R = \int 20 - 0.02x \, dx$
   $= 20x - 0.02\frac{x^2}{2} + C$
   $R(0) = 0 \rightarrow C = 0$
   $20x - 0.02\frac{x^2}{2}$
   $- 100$

   (a) \$18. $\int 20 - \int 0.02x \, dx =$

   (b) \$1000.

   [✓ (c)] \$1900.

   (d) \$2000.

3. *If* $y = \ln x^3$*, then* $\frac{dy}{dx} =$ $\frac{3}{x}$ $((3\ln x)' = \frac{3}{x}$

   [✓ (a)] $\frac{3}{x}$.

   (b) $\frac{1}{x^3}$. $\ln x^3 = \frac{3x^2}{x^3}$ $\frac{3\cancel{x^2}}{\cancel{x^3}2}$ $\frac{3}{x}$

   (c) $\frac{3}{x^3}$.

   (d) $\frac{3x^2}{lnx^3}$. $3/x$

   $\ln(x^2 + 2x + 1)$
   $\frac{1}{\quad}$
   $\ln x^3$
   $\frac{3x^2}{x^3}$

4. *If* $y = x2^x$ *then* $y'(x) =$ $(x)(2^x)\ln 2 + 2^x$
   $\ln 2$ $2^x(x\ln 2 + 1)$

   (a) $y'(x) = x2^x \ln 2$

   (b) $y'(x) = (x+1)2^x \ln 2$

   (c) $y'(x) = (x+1)2^x$

   [✓ (d)] $y'(x) = (x \ln 2 + 1)2^x$

$y = x2^x$ $y' =$

$x \cdot 2^x$

$+$

$2^x \cdot \ln 2$ $x2^x + 2x!$

2 $x \cdot 2^x.$

$y'\left(x \cdot 2^x \cdot \ln 2\right) + \left(2^x \cdot 1\right)$

10.5

2^{x-1} $\quad 2^{x-1} \cdot \ln 2 \cdot 1$

5. *The total number of units produced by a worker in t hours can be modeled by $p(t) = 27t + 12t^2 - t^3$, Find number of hours needed to reach point of diminshing returns.*

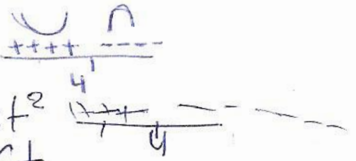
$P(t)' = 27 + 24t - 3t^2$

$P(t)'' = 24 - 6t = 0 \rightarrow 6(4 - t) = 0$, $t = 4$

$p(t)' = 27 + 24t - 3t^2$

$p(t)'' = 0 + 24 - 6t$

- *(a) 2 hours.*
- [✓ *(b) 4 hours.*]
- *(c) 8 hours.*
- *(d) None of the above*



6. *The Absolute maximum of the function $f(x) = 1 - x^2$, over $[-2, --1]$ is:*

$-2x = 0$

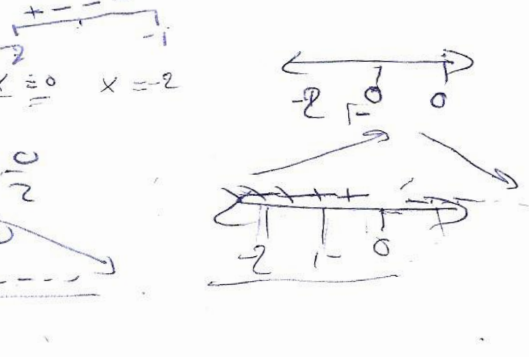
$x = 0$

$-2x = 0$ $\quad x = -2$

$\frac{1}{\ln 2} \cdot \ln x$

$\frac{1}{\ln 2} \cdot \frac{1}{x}$

- [✓ *(a) -2*]
- *(b) -1*
- [✓ *(c) 0*]
- *(d) 1*



7. *If $y = \log_2 x$, then $\frac{dy}{dx} =$* $\frac{1}{x \ln 2}$

$\frac{\ln x}{\ln 2}$

$\log_2 x$ $\quad \frac{\ln x}{\boxed{\ln 2}}$

$\log_2 x$

$\frac{\ln x}{\ln 2}$

$\frac{1}{x} \cdot \frac{1}{\ln 2}$

$\frac{\frac{1}{x}}{\frac{1}{\ln 2}}$

1

- *(a) $\frac{1}{x}$.*
- *(b) $\frac{\ln 2}{x}$.*
- *(c) $\frac{x}{\ln 2}$.*
- [✓ *(d) $\frac{1}{x \ln 2}$.*]

8. *If $y = x\ln x - x$, then $y'(x) =$* $\frac{\cancel{x}}{\cancel{x}} + \ln x - \cancel{x}$

$\cancel{x} \cdot \frac{1}{\cancel{x}} + 1 \cdot \ln x - 1$

$\cancel{1} + \ln x - \cancel{1}$

$= \boxed{\ln x}$

- *(a) $y'(x) = x^2 + \ln x - 1$*
- *(b) $y'(x) = \ln x - 1$*
- [✓ *(c) $y'(x) = \ln x$*]
- *(d) $y'(x) = x \ln x$*

Question 2. (4points). *If $f(x) = x^3 - 6x^2 + 4$, Find Interval/s where f concave down.*

$f(x) = x^3 - 6x^2 + 4$

$f'(x) = 3x^2 - 12x$

$f''(x) = 6x - 12$

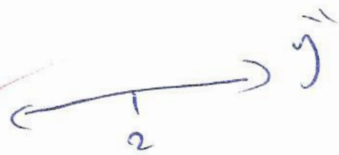
$3x^2 - 12x + 0$

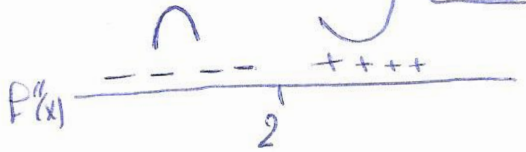
$6x - 12 = 0$

f''

$f''(x) = 0 \rightarrow 6(x - 2) = 0$ $\quad \boxed{x = 2}$

$\frac{6x}{6} = \frac{12}{6}$ $\quad \boxed{x = 2}$





$f(x)$ concave down : $(-\infty, 2)$

4

10 15

Question 3. (6points). *If the demand function* $P = \frac{10}{\sqrt{2q+5}}$ *dollars, x represents the number of units. Find*

1. *Find the elasticity of demand at* $q = 10$.
2. *What is the type of elasticity.*
3. *How will a Price decrease affect total Revenue.*

$$P = \frac{10}{\sqrt{2q+5}}$$

$$* \; E = \frac{-P}{q} \cdot \frac{dq}{dp}$$

$$q = 10 \rightarrow p = 2$$

1)

$$\left(P = 10(2q+5)^{-\frac{1}{2}} \right)'$$

$$1 = (10)(-\tfrac{1}{\cancel{2}})(2q+5)^{-\frac{3}{2}}(\cancel{2}\frac{dq}{dp})$$

$$\left(1 = \frac{-10}{\sqrt{(2q+5)^3}} \cdot \frac{dq}{dp} \right) \text{ at } q = 10$$

$$1 = \frac{-10}{\sqrt{(20+5)^3}} \cdot \frac{dq}{dp}$$

$$\sqrt{(20+5)^3}$$

$$10$$

~~$\frac{dq}{dp} = \frac{1}{4.294}$~~

$$\frac{\sqrt{(20+5)^3}}{-10} = \frac{-10}{-10}\frac{dq}{dp}$$

~~$10\frac{dq}{dp} = 10$~~

$$\boxed{\frac{dq}{dp} = -12.5}$$

$$E = \frac{+2}{\cancel{10}_{5}} \cdot +12.5$$

$$\boxed{E = 2.5} > 1$$

2) Elastic (at $q = 10$)

3) if the price decrease the total Revenue will increase

Elastic ——— العلاقة عكسية

6

0.36

Question 4. (5points). *Suppose the demand function for a product is given by:* $P = e^{-x}$ *, ,where p is the price per unit in dollars, and x is the number of units demanded,. Find*

$P = e^{-x} \rightarrow R(x) = e^{-x} \cdot x$

1. *Marginal Revenue function.*
2. *Price that Maximize Revenue.*

1 $MR = -xe^{-x} + e^{-x}$

2 $MR = 0$

$-xe^{-x} + e^{-x} = 0 \longrightarrow e^{-x}(1-x) = 0$

$\boxed{x = 1}$

MR ++++++ | − − − −
1

Unit that Maximize Revenue $\rightarrow \boxed{X = 1}$

Price that maximize Revenue $\rightarrow \approx 0.37$ \$

Question 5. (4points). *If the total cost function is given by* $C(x) = 810 + 0.1x^2$ *, Find number of units x will minimize the average cost.*

$C(x) = 810 + 0.1x^2$

$\overline{C(x)} = \frac{810}{x} + 0.1x \quad , \quad x \neq 0$

$\overline{C(x)}' = -\frac{810}{x^2} + 0.1 = 0$

$\frac{-810 + 0.1x^2}{x^2} = 0$

$-810 + 0.1x^2 = 0$

$x = +90 , \cancel{-90}$

$\overline{C(x)}''= \frac{810}{x^3} \Big|_{90}$

$\frac{810}{(90)^3} = 9 > 0$ Min

$\overline{C(x)}'$ − − − − | ++++
90

9

Number of units that minimize the average cost : $\boxed{x = 90}$

Question 6. (5points). *Find the Equation of the line tangent to the curve* $x^2y + e^{x+y} + 3x = 2y - 3$ *at the point* $(-1, 1)$.

2

~~$(x^2y + e^{x+y} + 3x = 2y-3)'$~~

~~$x^2y' + y\,2x + (1+y')e^{x+y} + 3 = 2y'$~~ ~~$(-1,1)$~~

~~$(1)y' + (1)(2)(-1) + (1+y')e^0 + 3 = 2y'$~~

~~$y' - 2 + 1 + y' + 3 = 2y'$~~

$2y'$

$x^2y + e^{x+y} + 3x = 2y - 3$

$2xy + x^2y' + (1+y')e^{x+y} + 3 = 2y'$ $(-1,1)$

$-2 + y' + (1+y')(1) + 3 = 2y'$

الحل لهذا السؤال خلف الورقة

Question 7. (4points). *If* $y = \ln \frac{\sqrt{5x-6}}{(2x-1)^3}$, *Find* $\frac{dy}{dx}$

$y = \frac{1}{2}\ln(5x-6) - 3\ln(2x-1)$

$y' = (\frac{1}{2})(\frac{5}{5x-6}) - (3)(\frac{2}{2x-1})$

$$\boxed{y' = \frac{5}{2(5x-6)} - \frac{6}{2x-1}}$$

6

Question 8. (10pcints). *Evaluate the following Integrals*

1. $\frac{-2}{-2}\int(1-2x)^{10}dx$

$$=-\frac{1}{2}\int -2(1-2x)^{10}\,dx$$

$$=\left(\frac{-1}{2}\right)\frac{(1-2x)^{11}}{11}+C$$

Or $1-2x=u$

$$\frac{du}{-2}=\frac{-2}{-2}dx$$

~~$\int(1-2x)$~~

$$\int u^{10}\frac{du}{-2}=\frac{1}{-2}\cdot\frac{u^{11}}{11}+C$$

2. $\frac{3}{3}\int\frac{x^2-2}{x^3-6x}dx$

$$=\frac{1}{3}\int\frac{(3x^2-6)}{(x^3-6x)}\,dx$$

$$=\frac{1}{3}\ln|x^3-6x|+C$$

3. $\int\frac{4x^2-4}{\sqrt{x^3-3x}}dx$

$$=\int(4x^2-4)(x^3-3x)^{-\frac{1}{2}}$$

$$=\frac{3}{3}\left(\int 4(x^2-1)(x^3-3x)^{-\frac{1}{2}}\right)$$

$$=\frac{4}{3}\int(3x^2-3)(x^3-3x)^{-\frac{1}{2}}=\frac{4}{3}\frac{(x^3-3x)^{\frac{1}{2}}}{\frac{1}{2}}=\frac{8\sqrt{x^3-3x}}{3}+C$$

4. $\int\frac{e^{3x}-e^{x}}{e^{5x}}dx$

$$=\int\left(\frac{e^{3x}}{e^{5x}}-\frac{e^{x}}{e^{5x}}\right)dx$$

$$=\left(\frac{-2}{-2}\right)\int e^{-2x}dx-\left(\frac{-4}{-4}\right)\int e^{-4x}\,dx$$

$$=-\frac{1}{2}\int -2e^{-2x}dx+\frac{1}{4}\int -4e^{-4y}dx$$

$-2x$ $-4x$

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